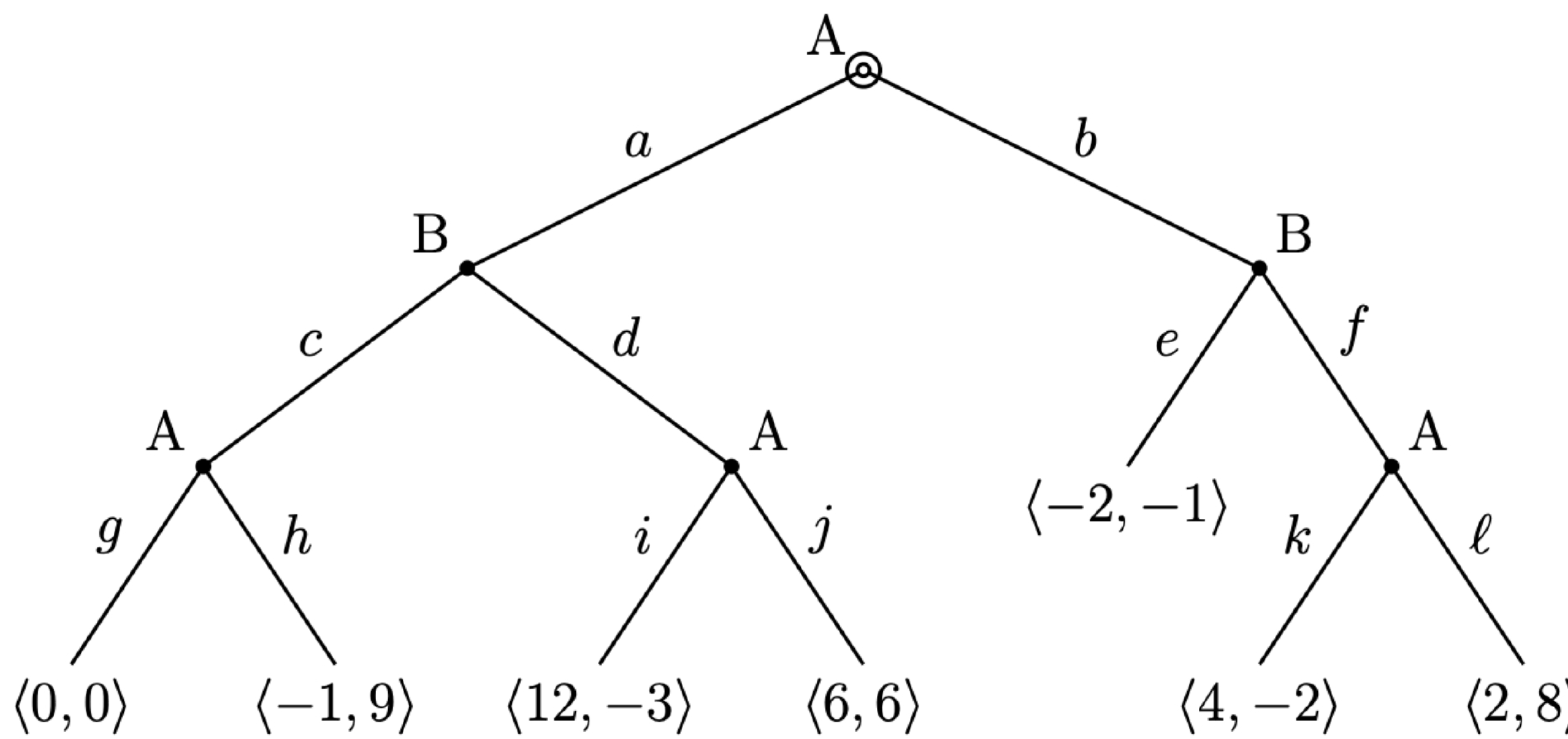
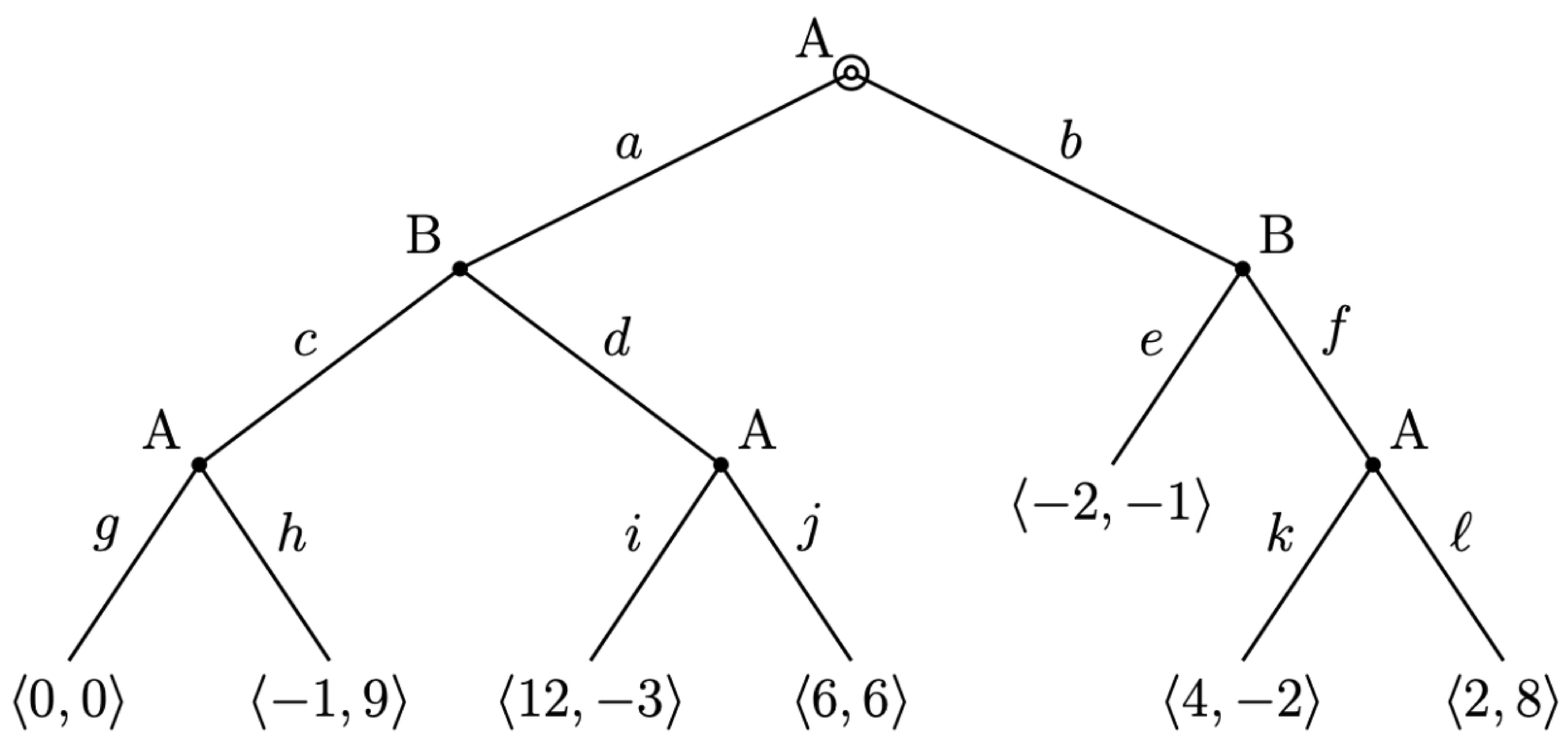
Extended Games Notes



Pure Strategies in an Extended Game

Starting at the top, one player at a time moves, and we progress down the tree to the end.



These CAN be analysed in "strategic form" (i.e. with matrices).

Player B

Player A

	c,e	c,f	d,e	d,f
→ a,g,i,k	<0,0>			
a,g,i,l				
⋮				
b,h,j,l				

A (pure) STRATEGY in an extended game is a "complete and contingent" plan of action for how to move at ANY point in the game.

That is, it assigns an ACTION to every node that the player controls.

Consider player A.

How MANY (pure) strategies do they have?

We need to specify an action at ALL FOUR of the nodes.

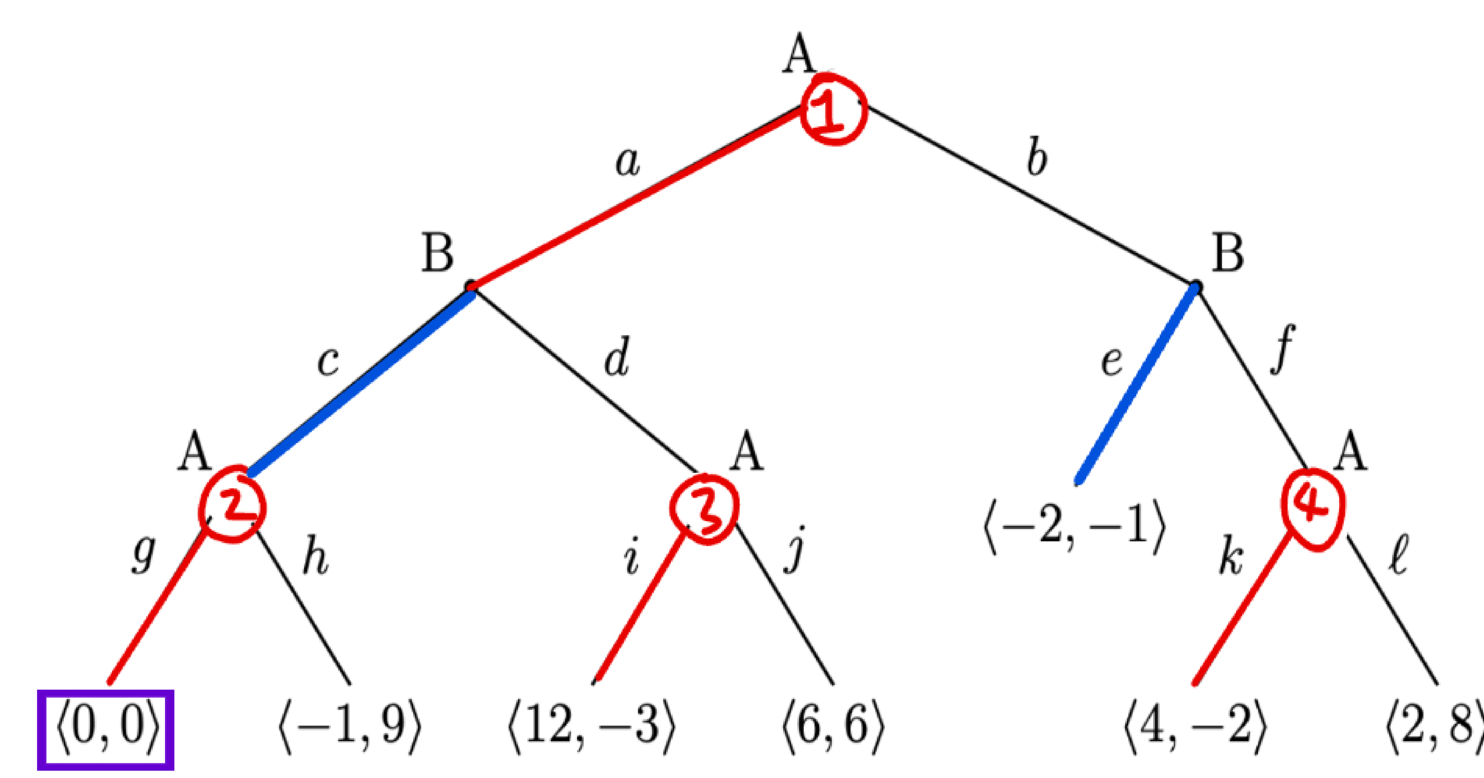
for example: (a,g,i,k)

n.b.: we still include "k", even though we never reach "4" when playing "a".

this makes things simpler overall! don't need to keep of which parts to include.

the number of strategies is 2x2x2x2 = 16

For player B, there are 2x2=4 choices.



From drawing the strategies on the tree, we can find the payoffs in the matrix.
BUT, this is actually a pretty bad way to analyse extended games (and NOT just because it's long...)

Recap. The most important concept in Game Theory is:

Nash Equilibria

A Nash Equilirium is a PAIR of strategies -- one for each player -- so that EACH is a "best response" to the strategy of the OTHER player.

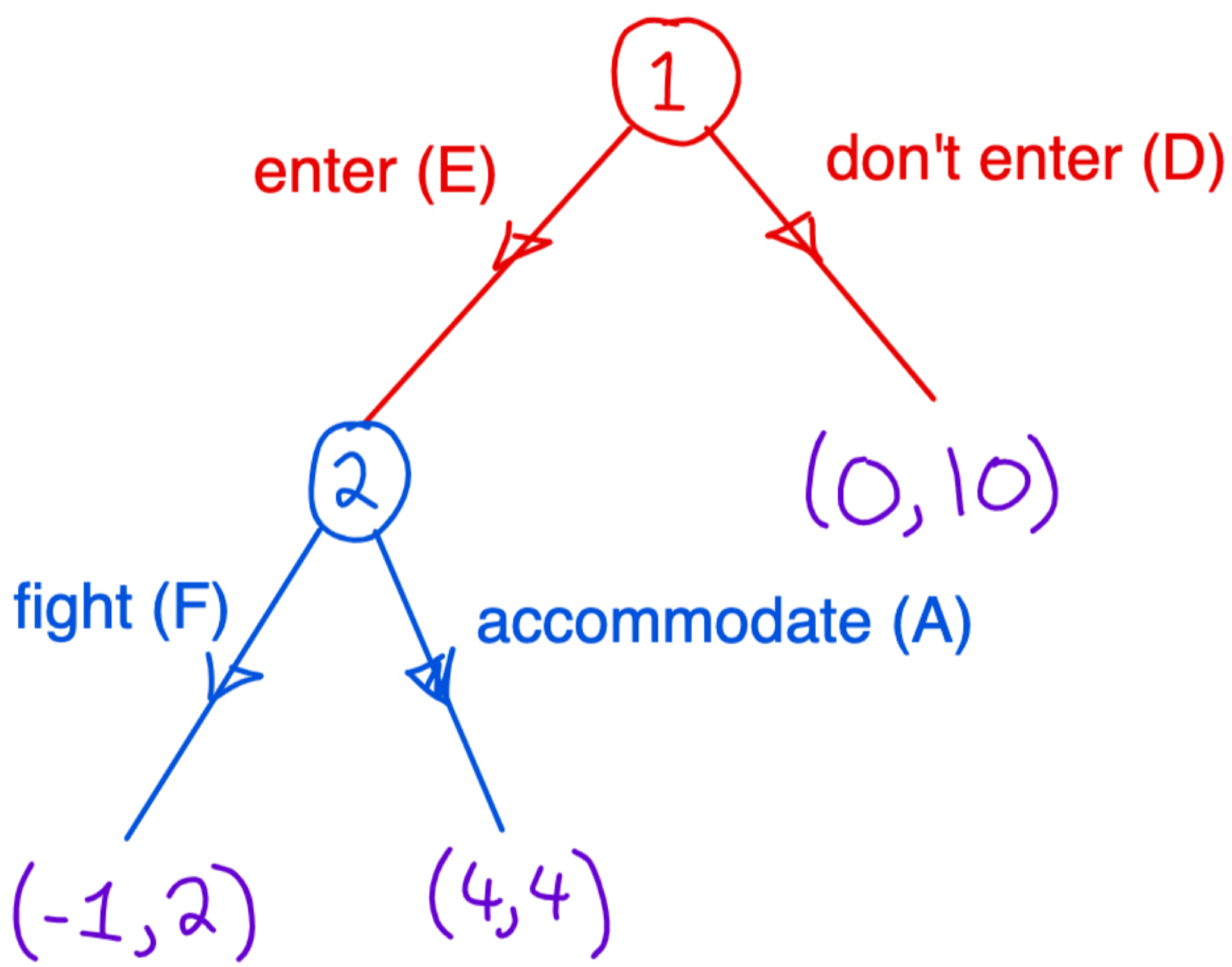
Alternatively: neither player can profit from CHANGING their strategy, assuming that the OTHER player continues with the SAME strategy.

A Classic Example

This well-known game shows why we should NOT analyse extended games by just making a matrix and looking for the Nash Equilibria.

Player 1 : Entrant (a firm which MAY enter an industry)

Player 2 : Encumbent (a firm which ALREADY dominates an industry)



We can write this game in NORMAL FORM using a matrix.

Player 1 \ Player 2	F	A
E	-1,2	4,4
D	0,10	0,10

We have two NE: (D,F) and (E,A). Let's examine these and check they really are NE.

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Let's examine these and
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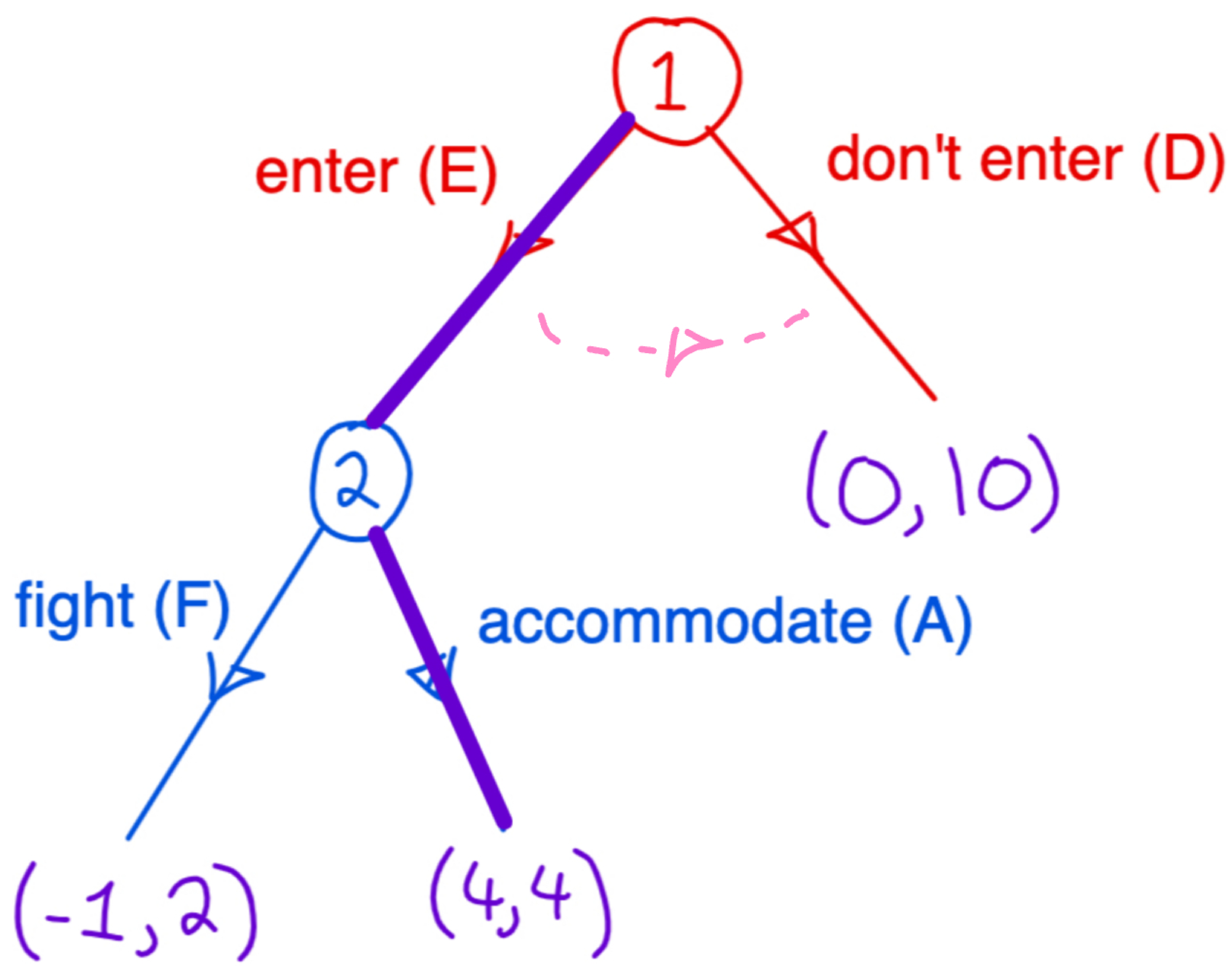
Enter, Accomodate



To check this is a NE, we need to check that neither player has an incentive to deviate to the other strategy, assuming the other player continues with the same strategy.

Let's first check for P1.

Their payoff with their current strategy, E, is 4.

If they deviate to "D", they get 0. ✓

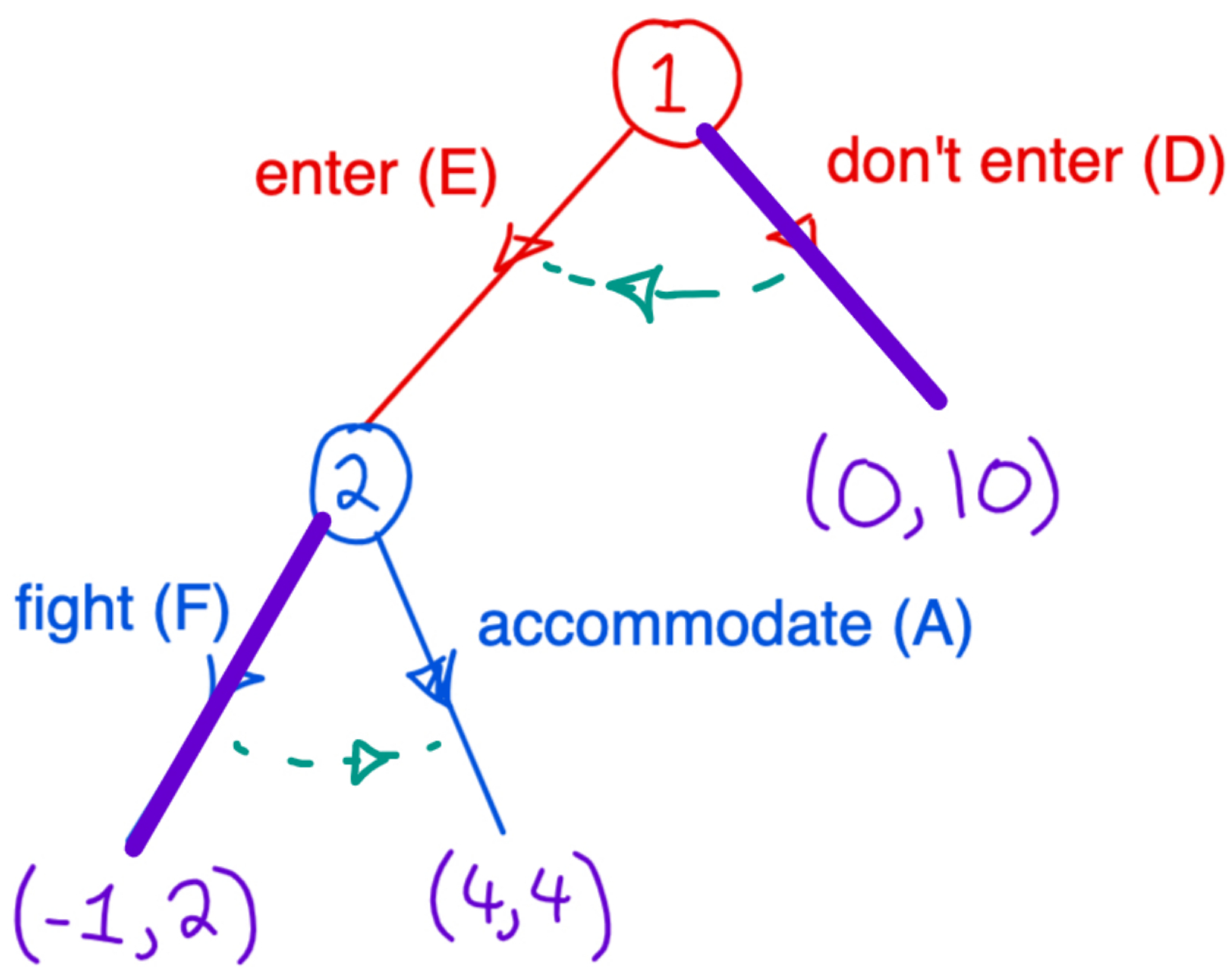
Let's check for P2.

Their payoff with their current strategy, A, is 4.

If they deviate to "F", they get 2.

Now let's consider the other equilibrium:

Don't Enter, Fight



To check this is a NE, we need to check that neither player has an incentive to deviate to the other strategy, assuming the other player continues with the same strategy.

Let's first check for P1.

With their current strategy, D, they get 0.

If they deviate, to E, they get -1. ✓

Let's check for P2.

With their current strategy, F, they get 10

If they deviate, to A, they still get 10! ✓

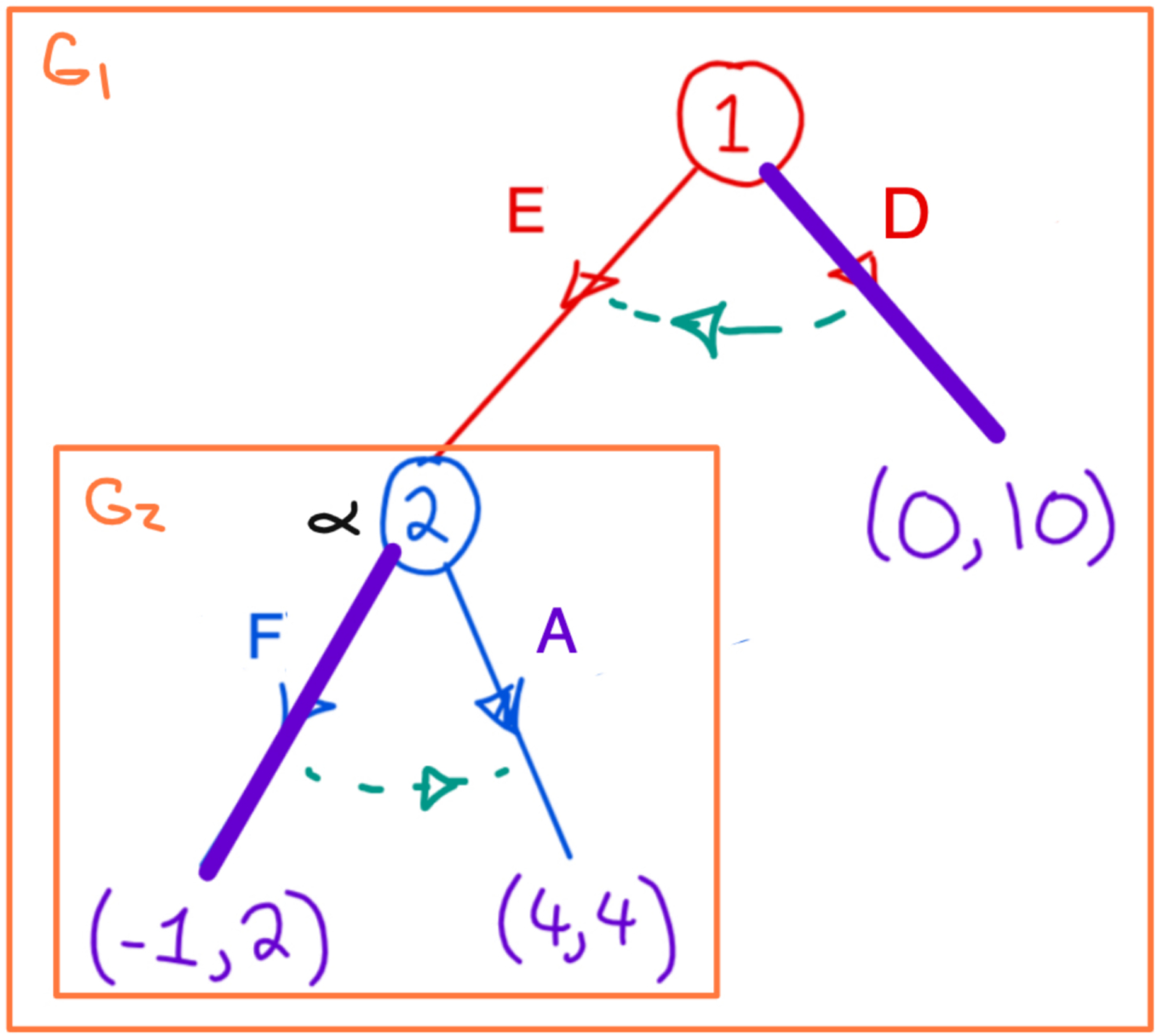
This is a Nash Equilibrium. But is there anything strange about it?
P2 seems to be behaving irrationally at their decision node. But this node is never reached, so it doesn't matter.

This is a "Non-credible threat" -- P1 avoids entering to avoid a fight/price war.
But "if they really did enter, P2 would have to accommodate"

informally, we'd like to say this, BUT it is NOT captured by the NE concept, since when P1 deviates, we have to hold P2's strategy fixed ...

Can we improve on the idea of NE, since it doesn't seem to capture what's going on this game? YES!

Subgames



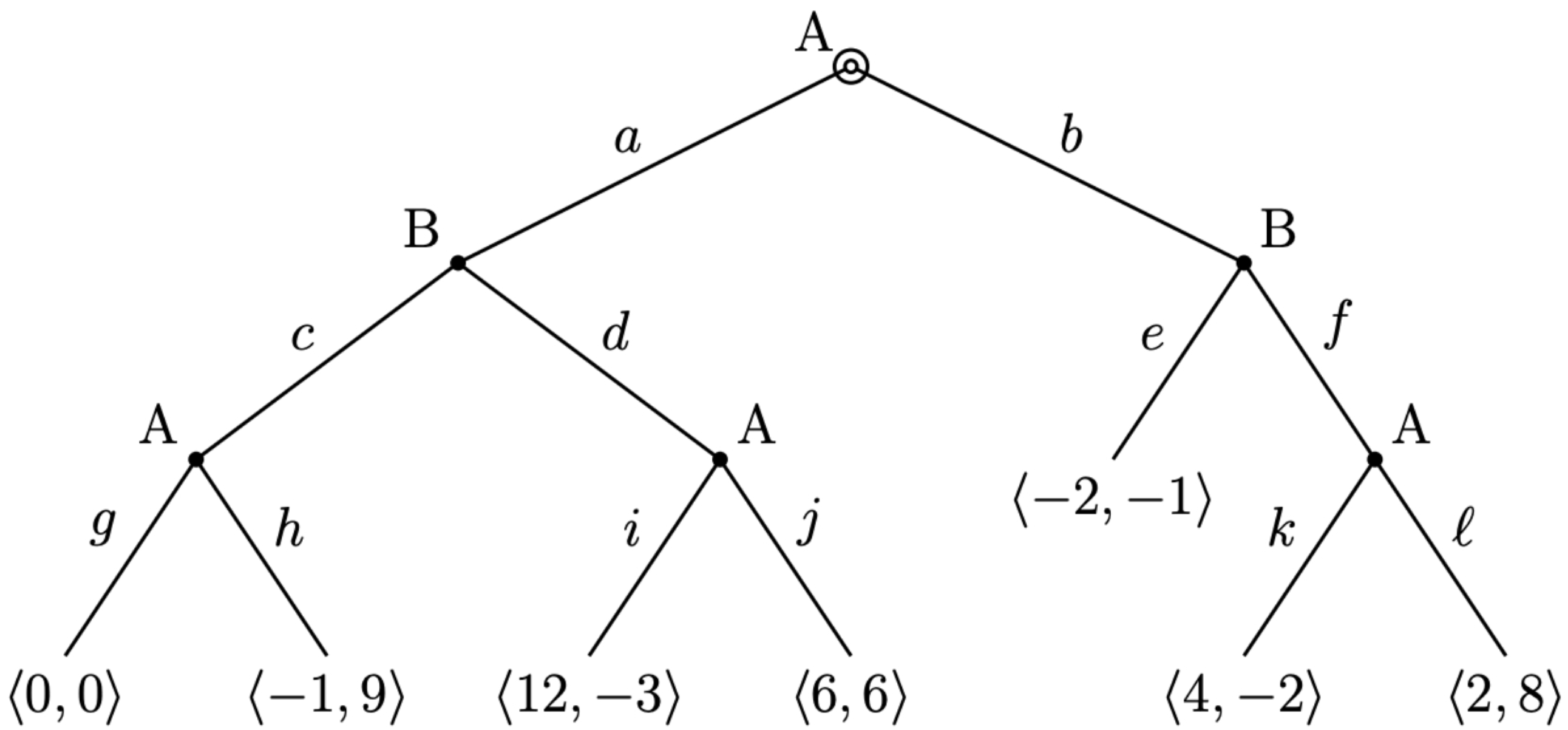
Key problem: at α, P2 seems to make an irrational move. Yet as we've seen, the strategy profile (D,F) is a genuine Nash Equilibrium.

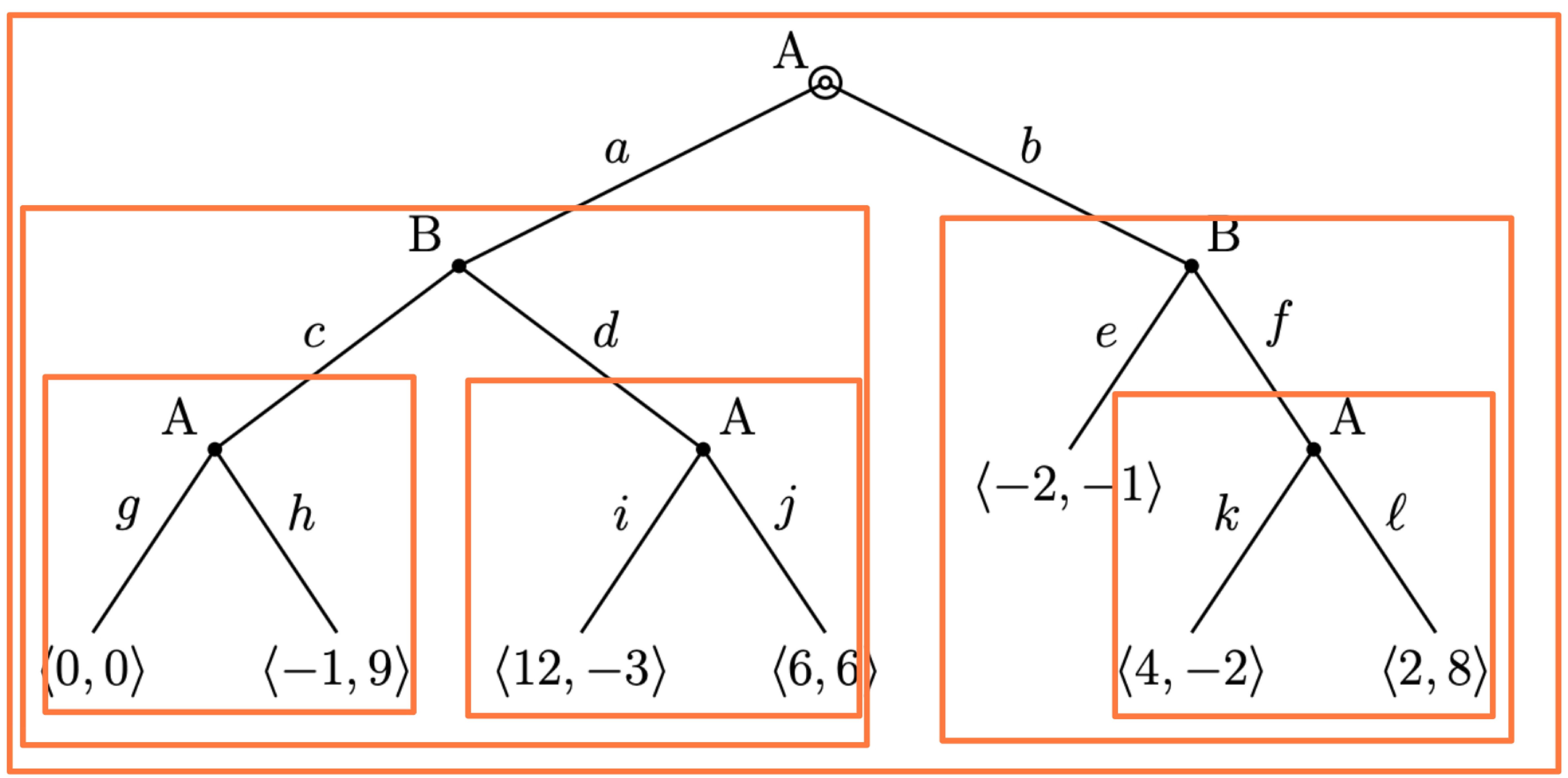
Solution: refine our definition of equilibrium to rule out this kind of case.

Key idea: the notion of a SUBGAME.

-- like a smaller game in its own right (though the "main game" is also a subgame
-- a decision node, together with the rest of the game, as played out from there
-- don't "cut across" information sets

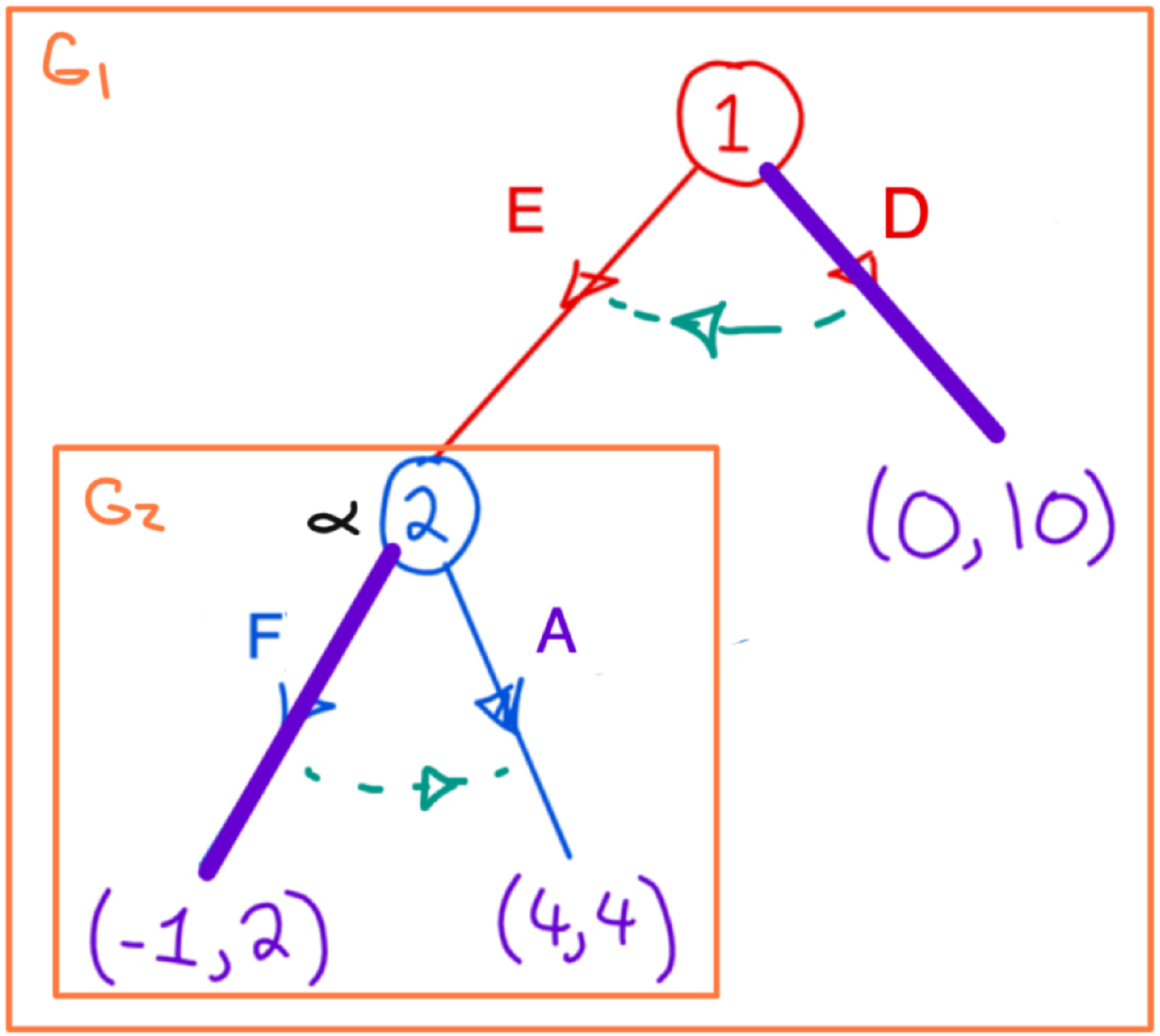
What are the subgames?





In total, we have 6 subgames.

Subgame-Perfect Equilibrium



A "strategy" for the main game is a complete and contingent plan of action for a player. It tells them what to play at ANY decision node they control.

(n.b.: even "off path" decision nodes that are not reached given all players' actions)

A strategy for the main game also tells us how to play in subgames. We may say it "induces" a strategy for each subgame.

if you know how to play in the main game, you can play in any subgame too!

A strategy profile is a "subgame-perfect equilibrium" if it induces a Nash Equilibrium strategy profile in ANY subgame.

That is, if the players play "the same moves" in any subgame, that will also give us a Nash Equilibrium in the subgame.

we should be playing NE in all subgames, too!

Consider the subgame G2.
P2 has strategy, F and A
P1 has strategy set ø

The only Nash Equilibrium here is for P2 to play A.

So, P2 must also play "A" in the main game in any SPNE.

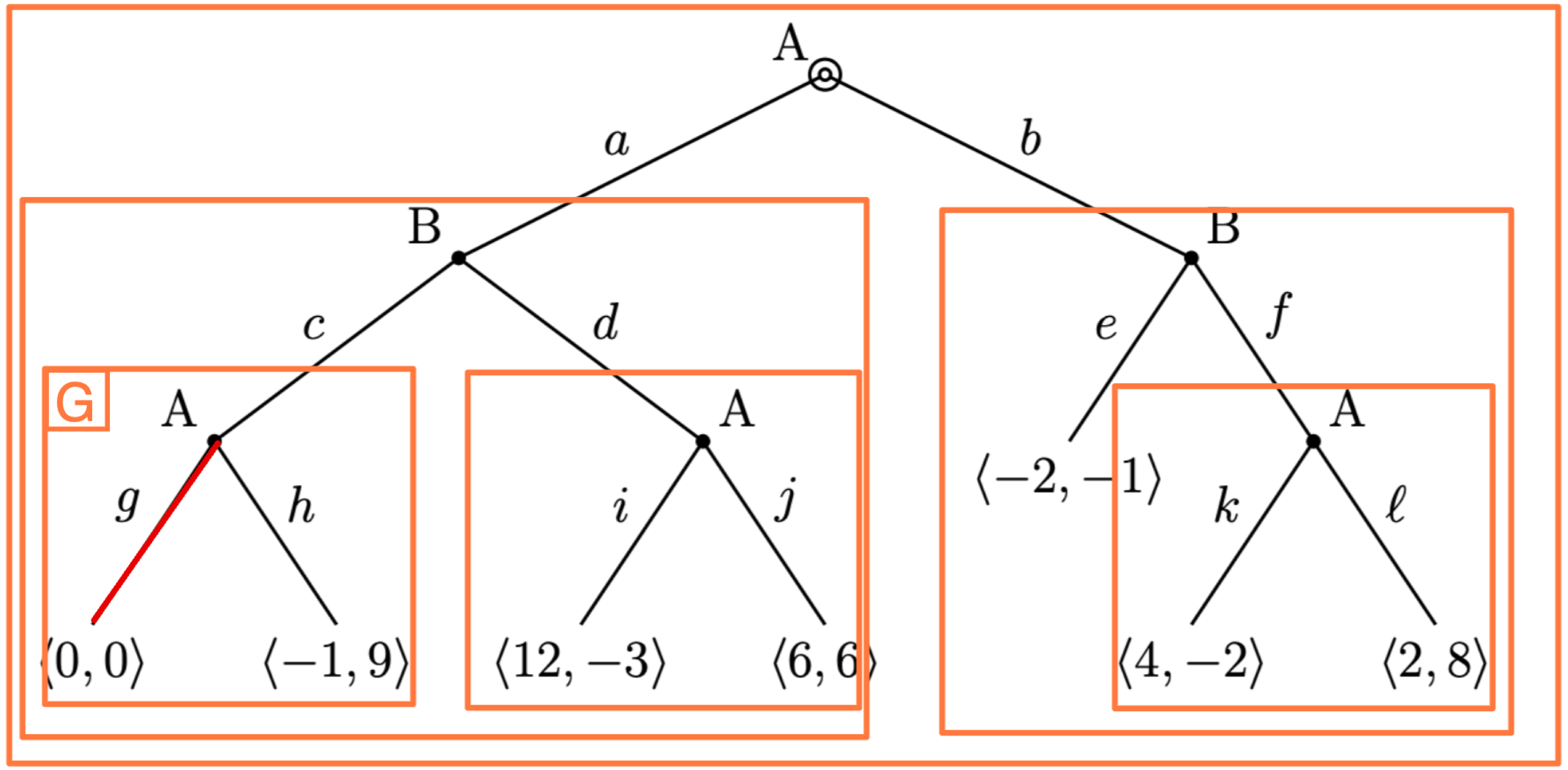
So in the main game,
(D,F) is <u>not subgame perfect.</u>

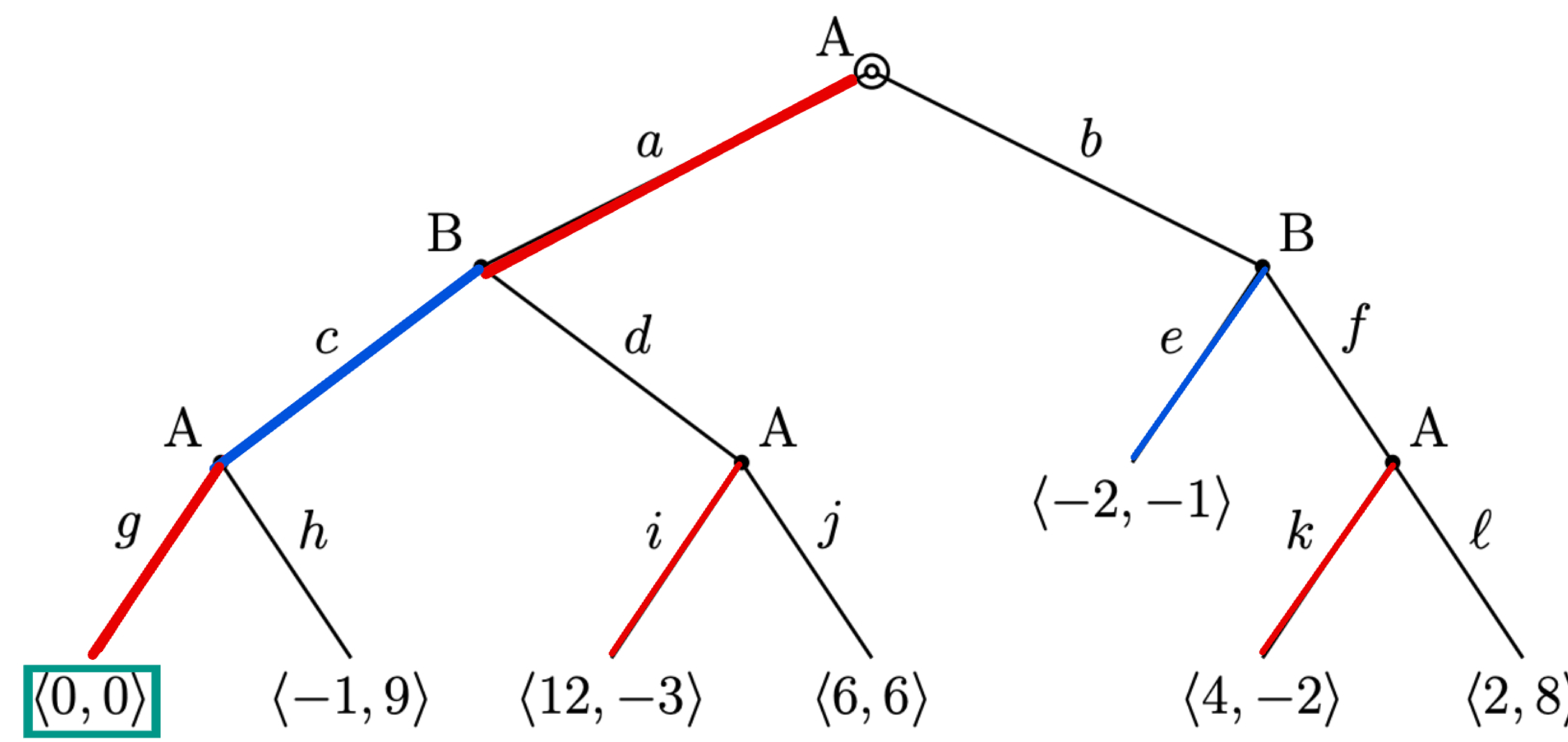
Finding Subgame Perfect Equilibria.

This is actually really easy, most of the time!

We call this "backwards induction". Basically:

-- start at the end; pick sensible moves
-- move backwards; anticipates moves worked out so far.





In the subgame "G", Player A must choose "g" for a NE.
So, their strategy must be (–, g, –, –) in the main game for a SPNE!

We can do this for all subgames where A moves last; then work backwards to see what B must be doing IN CASES WHERE A BEHAVES LIKE THIS (which must be the case in SPNE).

We found a unique SPNE:

(a,g,i,k) for "A", and (c,e) for "B".

n.b.: avoid the common mistake of only specifying the moves actually played! The "off-path" moves are important for the analysis too (to check for possible deviations).